Matrices: The arrangement of numbers in a brackets is called matrix.

e.g $[2]$, $[2, \ 3]$, $\begin{bmatrix} 5 & 6 \\ 1 & 3 \end{bmatrix}$ etc.

The horizental line of elements is called rows and the vertical line of elements is called Calumn.

Row is denoted by m and Column is denoted by n

Order of a matrix: The number of rows and column of a matrix is called order of a matrix, it is denoted by $m \times n$ or m by n. (Row x Column).

e.g $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ order = 2 by 1 or 2×1

$B = \begin{bmatrix} 3 & 4 \end{bmatrix}$ order = 1 by 2 or 1×2

General representation of a matrix: A matrix is represented generally as follows.

$A = [a_{ij}]$ where i = Row and j = Column.

e.g $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

So, $a_{11} = 1$, $a_{12} = 2$, $a_{21} = 3$, $a_{22} = 6$.

Types of Matrices: The following are the types of matrices.

① Row Matrix: A matrix which have only one row and more than one columns.

e.g $\begin{bmatrix} 1, & 2 & 5 \end{bmatrix}$, $\begin{bmatrix} 2 & 5 \end{bmatrix}$, $\begin{bmatrix} 1 & 3 & 4 & 6 \end{bmatrix}$ etc.

② Column matrix: A matrix which have only one column and more than one rows.

e.g $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 9 \\ 3 \\ 1 \\ 6 \end{bmatrix}$ etc.

③ Square matrix: A matrix which have same number of rows and columns.

e.g $[2]$, $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, $\begin{bmatrix} 2 & 3 & 1 \\ 5 & 7 & 8 \\ 0 & 5 & 7 \end{bmatrix}$ etc.

④ Rectangular matrix: A matrix which have different number of rows and columns. ~~and also~~

e.g $\begin{bmatrix} 1 & 5 & 3 \\ 2 & 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 5 \end{bmatrix}$ etc.

⑤ Null or Zero matrix: A matrix in which all the elements are equal to zero.

e.g $[0]$, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ etc.

Diagonal matrix: A square matrix in which at least one element of ~~a~~ principle diagonal is non-zero and all other elements are equal to zero.

e.g $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$ etc.

Scalar matrix: A square matrix in which ~~all~~ the elements of principle diagonal are equal and all others elements are equal to zero.

e.g $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ etc.

Unit or Identity matrix: A square matrix in which the elements of principle diagonal are equal to 1 and all other elements are equal to zero.

e.g $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $[1]$. etc.

Transpose of a matrix: If we change columns of ~~a~~ a matrix in rows or rows in columns we get the transpose. It is denoted by A^t.

i.e $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$, $A^t = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$.

Symmetric Matrix: A matrix A is said to be symmetric if $A^t = A$.

e.g $A = \begin{bmatrix} 1 & 5 \\ 5 & 3 \end{bmatrix}$, $A^t = \begin{bmatrix} 1 & 5 \\ 5 & 3 \end{bmatrix}$ so, $A = A^t$.

Skew symmetric: A matrix A is said to be skew symmetric if $A^t = -A$.

e.g $A = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} \Rightarrow A^t = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

$A^t = -\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$ so, $A^t = -A$.

Addition of matrices: Two matrices are said to be conformable for addition if their order are same.

e.g $A = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 5 \end{bmatrix}$

$A + B = \begin{bmatrix} 3 & 7 \end{bmatrix}$ etc.

Same case for subtraction.

Q: Add the following matrices.

i) $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}$

ii) $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 6 & 7 \end{bmatrix}$

iii) $A = \begin{bmatrix} 1 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 3 \end{bmatrix}$

iv) $A = \begin{bmatrix} 1 & 5 & 0 \\ 3 & 7 & 8 \\ 6 & 9 & 10 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 5 \\ 0 & 6 & 7 \\ 1 & -8 & 3 \end{bmatrix}$

Q: Subtract A from B in the above. ~~equ~~ question.

Multiplication of matrices: Two matrices A and B are said to be conformable for multiplication.

if (i) AB Number of columns of A = number of rows of B.

(ii) BA No of columns of B = No of rows of A.

e.g. $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 \end{bmatrix}$ AB is possible

Also BA is possible.

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \end{bmatrix}$

then AB is not possible and BA is possible

Q: Find the product of the following Matrices if possible.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$D = \begin{bmatrix} 3 & 6 \end{bmatrix}$

Find AB, BA, AC, CA, BD, DB, DC, AD

~~Common~~

Scalar Multiplication: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

$2A = \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$ where 2 is a scalar Multiple.

Generally $KA = \begin{bmatrix} Ka & Kb \\ Kc & Kd \end{bmatrix}$.

Determinant: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a square matrix then $|A| = ad - cb$ is called the determined.

e.g $A = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix} \Rightarrow |A| = 3 - 10 = -7$

Determinant of 3x3 Matrix :

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ then

$$|A| = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}.$$

$$|A| = a(ai - hf) - b(di - gf) + c(dh - ge).$$

Example: Find $|A|$ if $A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 8 \\ 1 & 0 & 3 \end{bmatrix}$

$$|A| = 1\begin{vmatrix} 7 & 8 \\ 0 & 3 \end{vmatrix} - 2\begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} + 3\begin{vmatrix} 5 & 7 \\ 1 & 0 \end{vmatrix}$$

$$|A| = 1(21 - 0) - 2(15 - 8) + 3(0 - 7)$$

$$|A| = 21 - 2(7) + 3(-7)$$

$$|A| = \cancel{21} - 14 - \cancel{21} = -14$$

Find the determinant of the following Matrices.

$A = \begin{bmatrix} 0 & 5 & 3 \\ 1 & 2 & 0 \\ 3 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 2 \\ 3 & 5 & 4 \end{bmatrix}$. $C = \begin{bmatrix} 2 & 5 & 3 \\ 1 & 3 & 7 \\ 5 & 3 & 1 \end{bmatrix}$

Singular Matrix: A square matrix A is said to be singular if $|A| = 0$

e.g. $A = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & 6 \\ 3 & 9 \end{vmatrix} = 18 - 18 = 0.$

Non-singular Matrix. A square matrix A is said to be Non-singular if $|A| \neq 0$.

e.g. $A = \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}$. $|A| = \begin{vmatrix} 1 & 5 \\ 3 & 7 \end{vmatrix}$

$|A| = 7 - 15 = -8$

Q: Check the following matrices are singular or Non-singular.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 3 \\ 8 & 12 \end{bmatrix}$

$D = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 9 & 6 \\ 1 & 3 & 8 \end{bmatrix}$. $e = \begin{bmatrix} 2 & 0 & 5 \\ 3 & 1 & 6 \\ 1 & -2 & 3 \end{bmatrix}$

Adjoint of Matrix of A is a square matrix and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then adjoint of $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Similarly in 3×3 matrix

$$\text{adjoint of } A = \begin{bmatrix} \text{Cofactors of} \\ A \end{bmatrix}^T$$

Minor of a Matrix: We take an example.

Let $A = \begin{bmatrix} 1 & 5 \\ 3 & 8 \end{bmatrix}$

Minor of 1 = 8, Minor of 5 = 3

Minor of 3 = 5, Minor of 8 = 1

So, Minor Matrix of $A = \begin{bmatrix} 8 & 3 \\ 5 & 1 \end{bmatrix}$

Minor of elements: Let A be a square matrix of order n. The minor of the element a_{ij} is denoted by M_{ij} and it is the determinant of $(n-1) \times (n-1)$ matrix.

e.g. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$.

Minor of $a_{ij} = M_{ij}$

Minor of $a_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{32}a_{23}$

Minor of $a_{12} = M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = a_{21}a_{33} - a_{31}a_{23}$

Cofactor and Minor of an element:

The following formula is used to find the Minor and Cofactor of an element.

Cofactor of $a_{ij} = (-1)^{i+j} M_{ij}$

Cofactor of $a_{11} = (-1)^2 M_{11} = +M_{11}$

Cofactor of $a_{12} = (-1)^3 M_{12} = -M_{12}$

Cofactor of $a_{13} = (-1)^4 M_{13} = +M_{13}$

" " $a_{32} = (-1)^5 M_{32} = -M_{32}$

" " $a_{31} = (-1)^4 M_{31} = +M_{31}$

" " $a_{33} = (-1)^6 M_{33} = +M_{33}$